A Vector Tabu Search Algorithm with Enhanced Searching Ability for Pareto Solutions and its Application to Multiobjective Optimizations

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A vector tabu search algorithm encapsulating a new updating mechanism for current state and a directed search phase is proposed to enhance its searching ability for Pareto solutions. The new updating mechanism considers quantitatively both the number of improved objectives and the amount of improvements in a specified objective. The directed search phase uses some desired directions, *a priori* knowledge about the objective space, as the moving direction to efficiently find improved solutions without any gradient computation procedure. The numerical results on a high frequency inverse problem are reported to demonstrate the *pros* and *cons* of the proposed algorithm. It is observed that the proposed vector tabu search method outperforms its ancestors in both convergence performance and solution quality.

***Index Terms*—Design optimization, inverse problem, multiobjective, tabu search algorithm.**

I. Introduction

IN ENGINEERING inverse and optimal problems, it is not uncommon to ask a designer to satisfy several seemingly conflicting objectives simultaneously to meet a set of tradeoffs among different criteria. Generally, the final solutions of such a design problem are a set of compromises of different objectives called Pareto optimal solutions. Consequently, an acceptable multi-objective optimizer should have the ability to find as many Pareto optimal solutions as possible, and these solutions should also be as uniformly distributed as possible. To achieve these two ultimate goals, a huge amount of efforts have been devoted to the advancement of evolutionary algorithms (EA), because of their suitability in finding multiple solutions in a single run [1]-[3]. Nowadays, evolutionary algorithms have become the paradigms for solving multiobjective design problems.

However, it is well-known that EAs normally require a large amount of function evaluations, due to their slow convergence rate, in order to generate a suitable finite size approximation of the set of interests [4]. Moreover, the dominant concept is commonly used in EAs to assign the fitness value of an individual. Nevertheless, such approach may only determine qualitatively the relationship of dominances and may not measure, quantitatively, the number of improved objectives [5]. The amount of improvements in a specified objective may not be quantified either. Also, very few efforts have been devoted to address the balance between the conflict for the convergence towards the Pareto front, and the requirement to maintain diversities in the searched Pareto optimal solutions [6]. In an attempt to addressing these deficiencies of existing multiobjective EAs, a specially defined metric measure is introduced, and a new updating mechanism for current state using this metric is designed and encapsulated into a vector tabu search algorithm [7]. Also, a directed search phase is integrated as a refinement searching in order to efficiently find promising solutions.

II. A Enhanced Vector Tabu Search Algorithm

The details about the vector tabu search method are refereed to [7] and the references therein. This section will describe only the new proposals to enhance the searching ability for Pareto solutions of the algorithm.

A. Updating Mechanism of Current State

The updating mechanism for current state in a scalar tabu search algorithm is that the "best" solution of the feasible moves generated in neighbors of the current state is selected as the new current one. Nevertheless, the optimal solutions of a multi-objective design problem are not unique but a set of tradeoffs among different objectives. This feature will give rise to a dilemma in selecting the "best" feasible move when updating the current states since the fitness values of different Pareto optimal solutions are the same. To address this issue, and moreover to further consider both the number of improved objectives and the amount of improvements in a specified objective for different Pareto solutions, an improvement metric is introduced. For a neighbor solution, x_m, the proposed metric is precisely given by

$$\varepsilon_{improve}(x_m) = \sum_{\substack{j=1 \\ j\neq k}}^{R} \sum_{l=1}^{k} sign[f_l(x_j) - f_l(x_m)]\{1 + \frac{f_l(x_j) - f_l(x_m)}{(f_l)_{\max} - (f_l)_{\min}}\} \tag{1}$$

$$sign(x) = \begin{cases} 1 & (x>0) \\ 0 & (\text{otherwise}) \end{cases} \tag{2}$$

where R is the number of neighborhood solutions, k is the number of objectives.

Using this accumulation factor to 'penalize' the fitness value of the neighborhood solution x_m yields the following fitness assignment formula

$$f_{fit}(x_m) = r f_{fit}^{nor}(x_m) + (1-r)\varepsilon_{\text{improve}}(x_m) \tag{3}$$

where, $f_{fit}^{nor}(x_m)$ is the normally defined fitness of x_{m} using the dominance concept, r is a random parameter uniformly generated in [0,1].

The best move among all neighborhood solutions in terms of their fitness values as given in (3) will be selected as the new current state. It should be pointed out that some "most" likely but not exactly Pareto solution may be selected as the new current state under the proposed updating mechanism of current states, guaranteeing the diversity of the algorithm.

B. A Directed Search Phase

To steer the searches in a desired direction to find improved solutions efficiently, a directed search method is proposed and integrated as a refinement searching phase. To start with, the "Pareto front" is firstly approximated using the solutions in the External Archive. To provide the desired search directions for the directed search using *a priori* knowledge about the objective space, it is proposed to construct a hypercone centered in the solution in question, x_j, and identify the closest point, x_h, on this hypercone, to x_j; and the desired searching direction, d_i, is then readily defined by connecting the two points in the objective space. To seek the corresponding searching direction, v, in the parameter space for d_i^k, the following equation is solved [8]

$$\lim_{t\to 0}\frac{f_l(x_j+tv)-f_l(x_j)}{t}=d_i^l\ (l=1,\ldots,k)\,, \tag{4}$$

where f_l is the l^{th} objective. (4) can be mathematically transformed to:

$$J(x_j)v=d_i\,, \tag{5}$$

where $J(\cdot)$ is the Jacobian of the objective vector F, and moreover,

$$\begin{aligned}&F(x)=(f_1,f_2,\cdots,f_k)\\&x=(x_1,x_2,\cdots,x_n)\end{aligned} \tag{6}$$

Since $n \gg k$, (5) is highly underdetermined, and the solution with the least 2-norm is viewed as the greedy direction, resulting in

$$v^+=J(x_j)^+d_i\,, \tag{7}$$

where $J(x_j)^+$ is the pseudo inverse of $J(x_j)$.

The refinement search around solution x_j in direction v is easily implemented using

$$x_{new}=x_j+tv\,, \tag{8}$$

where t is a small step size vector, and t_i is randomly selected from $[0, 0.05\Delta_i]$(Δ_i is the dimension size of the i^{th} variable).

III. Numerical Examples

To evaluate the performances of the proposed method, it is numerically experimented on both high and low frequency inverse problems. Due to space limitations, only numerical experiments on a high frequency inverse problem are reported.

In this case study, the optimal design goal is to synthesize a non-uniformly spaced antenna array, as shown in Fig. 1, to produce a desired field pattern. A two objectives optimal problem is thus formulated as [7]

$$\begin{cases}\min f_1\\ \min f_2\end{cases} \tag{9}$$

where, MSL*Ldesired* is the desired Maximum SideLobe Level (MSLL), and,

$$f_1=\sqrt{\sum_{i=1}^{N}[f_{desired}^{norm}(\theta_i)-f_{designed}^{norm}(\theta_i)]^2\Big/\sum_{i=1}^{N}[f_{desired}^{norm}(\theta_i)]^2} \tag{10}$$

$$f_2=MSLLdesired \tag{11}$$

where, $f_{desired}^{norm}(\theta_i)$ is the value of the normalized desired radiation pattern at the sampling point θ_i, $f_{designed}^{norm}(\theta_i)$ is the value of the radiation pattern produced by a designed array of M elements, N is the number of total sampling points. The field pattern is computed by using an analytical solution [7].

To evaluate the average performance of the proposed algorithm, it is run randomly and independently 10 times, and the average number of iterations is 29465, this is compared to 34568 of the original vector tabu search algorithm of [7]. Moreover, to compare the quality of the final solutions, the 135 searched Pareto solutions of the proposed (Proposed) algorithm in a typical run are compared to 121 ones of the original tabu (OTabu) search method in also an arbitrary run. It is observed that:

(1) 8 out of the total 135 Pareto solutions of the Proposed are dominated by at least one of the Pareto solutions of OTabu.

(2) 39 out of the total 121 final solutions of the OTabu are dominated by at least one solution of the Proposed.

Obviously, the proposed vector optimal tabu search algorithm outperforms its ancestors in both convergence performance and solution quality.

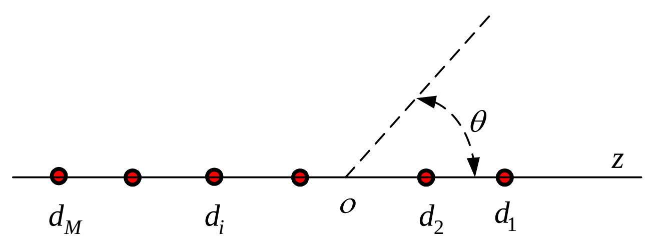

Fig. 1 The configuration of *M*-element linear arrays placed on the *z*-axis.

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